Universal tangent formula

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Contents

1 Introduction **2**

2 Methodology **3**

3 Algebraic examples **4**

4 References **5**

1 Introduction

Considering that calculus is a pivotal portion of VCE Mathematical Methods, it is crucial for students to explore efficient methods and strategies when working with calculus. For this reason, the focus of this document is on the concept of the tangent line. A tangent line is a line that lies on a function such that it matches the rate of change of the function at that value of x, as well as intersecting the graph at that value of x. Commonly, students are asked to solve for the tangent line, given a function or the derivative of a function, which involves considerable amounts of working out to perform. However, this document covers a simpler and efficient means of doing so, which has the benefit of faster times as VCE Mathematical Methods is drag.

2 Methodology

Consider a function $f(x)$, where $x \in \mathbb{R}$. The derivative of $f(x)$, giving $f'(x)$, provides the gradient of the tangent line by substituting the desired value of x in place of x, given that the value of x provides a valid point that lies on $f'(x)$. For example:

$$\text{A point on } f'(x) \text{ can be written as: } (a, f(a)), a \in \mathbb{R} \tag{1}$$

Thus, the tangent line at $x = a$ can be written as:

$$y = f'(a)x + c \tag{2}$$

However, the value of c must be determined in terms of a and $f(a)$ to formulate a universal equation of the tangent line.

As variables y and x are related through $f'(x)$, a point on $f(x)$ can be determined to evaluate c.

Recall $x = a$, where a is a possible x coordinate. Thus, the y value at $x = a$ can be determined through substituting a into $f(x)$.

$$x = a \text{ where } a \in \mathbb{R},\ \therefore (a, f(a)) \text{ lies on } f(x) \tag{3}$$

Going back to equation (2), the tangent line with c yet to be determined, the valid coordinate set of (3) can be substituted into the equation (2) as shown:

$$y = f'(a)x + c \rightarrow f(a) = f'(a)a + c$$

Simple transposition using algebra such that the subject is c indicates that:

$$f(a) = f'(a)a + c \tag{4}$$

$$\therefore f(a) - f'(a)a = c \tag{5}$$

Substitute (5) into (2)

$$y = f'(a)x + f(a) - f'(a)a \tag{6}$$

Thus the tangent line for any function that is differentiable is given by (6)

3 Algebraic examples

Example 1: Consider the function $f(x) = x^2$. The derivative of x^2 is $2x$, which can be represented as $f'(x)$. Suppose the tangent line at $(4, 16)$ is to be determined. As the desired x value, the function, and its derivative are known, the values can be substituted into equation (6).

Thus, $y = 8x - 16$

Example 2: Consider the function $f'(x) = \sin(x)$. The derivative of $\sin(x)$ is $\cos(x)$, which can be represented as $f'(x)$. Suppose the tangent line at $(\frac{\pi}{6}, \frac{1}{2})$ is to be determined. As the desired value of x, $\frac{\pi}{6}$, can be substituted in $f(x)$ and $f'(x)$ to return the values $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$, respectively, the tangent line can be evaluated through substitution into equation (6).

Thus, $y = \frac{\sqrt{3}}{2}x + \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\pi}{6}$
Which simplifies to: $y = \frac{\sqrt{3}}{2}x + \frac{6-\pi\sqrt{3}}{12}$

4 References

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